Optimum Polar Satellite Networks for Continuous Earth Coverage

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The problem of providing continuous coverage of portions of the earth's surface by means of polar satellite networks is considered, and an optimum arrangement is proposed in which the motion of satellites in one orbital plane is synchronized with that of the satellites in adjacent planes. Relations are developed between the numbers of satellites, their relative positions, and the required minimum latitude of continuous coverage. The method of solution of these relations is outlined, and an example is presented. The resultant number of satellites required for this case as a function of altitude is compared with the number required for a symmetric, nonsynchronous set of satellite orbits.

Nomenclature

h = orbital altitude
n_1 = number of orbital planes
n_2 = number of satellites per orbital plane
N = total number of satellites in orbit, $N = n_1 \times n_2$
R = radius of the earth, $R = 3440$ naut miles
S = slant range
α, β = incremental longitudes, defined by Fig. 9
δ = scan angle of satellite, defined by Fig. 1
ϵ = incremental longitude between counter-rotational satellite planes
θ = geocentric angle of coverage
$\lambda_1, \lambda_2, \lambda_3$ = latitudes of three satellites, defined by Fig. 9
λ_S = minimum latitude observed continuously by symmetric network
λ_T = minimum latitude observed continuously by nonsymmetric network
σ = minimum angle of visibility, defined by Fig. 1
ϕ = incremental longitude between co-rotational satellite planes
ψ = great circle arc, half-width of band of continuous coverage

Recent emphasis on the use of polar satellite networks for continuous coverage of portions of the earth's surface has led to studies of minimum satellite requirements to satisfy these needs (1–3).[3] However, Gobetz (2) and Vargo (3) consider only the problem of obtaining complete global coverage. Lüders (1) describes a network in which satellites are symmetrically placed over the surface of the earth and develops a technique by which the satellite requirement for specified altitude and area of coverage may be determined. However, this method requires a high degree of redundancy of coverage. The present paper proposes an optimum arrangement in which the motion of satellites in one orbital plane is synchronized with that of the satellites in adjacent planes and compares the satellite requirements based on this scheme with those determined from Ref. 1.

Network Design Concepts

Coverage Requirements

To provide continuous coverage of a specified area of the earth's surface, it is necessary to arrange the satellite network so as to obtain complete zonal coverage between the latitude planes bounding the extremes of the area of interest. This is necessary in order to maintain coverage of the specified area as it rotates diurnally. All of the surface area above the lower latitude boundary (referred to as the minimum latitude of continuous coverage) is automatically covered. since for polar networks all orbits converge at the poles, Therefore, only the minimum latitude of continuous coverage need be considered in the design of polar networks.

From geometric relationship of Fig. 1, the altitude h and slant range S may be expressed as a fraction of the earth's radius R as follows:

$$\frac{h}{R} = \frac{\cos\sigma}{\cos(\sigma + \theta)} - 1 \qquad [1]$$

$$\frac{S}{R} = \frac{\sin\theta}{\cos(\sigma + \theta)} \qquad [2]$$

In order to maintain consistence with the results of Ref. 1, a constant σ assumption (representing a minimum permissible value) will be used in this paper to define required satellite altitude.

The geocentric coverage angle θ required for the specified coverage is dependent on the following factors: n_1, the number of orbital planes; n_2, the number of satellites per orbital plane; λ_T, the minimum latitude of continuous coverage; and the method of satellite sequencing (e.g., Eqs. [A1a–A8] of the Appendix, for the case of synchronized sequencing). In addition, for a given total number of satellites N, there are generally a number of workable combinations of n_1 and n_2, all of which have different values of θ. However, one particular combination will always yield the smallest value of θ required for a given N and λ_T, representing the minimum number of satellites for a given coverage requirement.

Method of Sequencing

Symmetrical networks

Design concepts for satellite networks having n_1 orbital planes symmetrically arranged every π/n_1 degrees, as shown in Fig. 2, have been developed in Ref. 1. In each of these orbital planes there are n_2 satellites, symmetrically arranged every $2\pi/n_2$ degrees apart, and each satellite is capable of viewing a circular area of "radius"[4] θ on the earth's surface.

To provide a band of continuous coverage for each orbital plane, there must be a sufficient number of satellites such that the areas of detection of adjacent satellites in the same plane overlap. Planes parallel to the orbital plane and passing through the common points of the circular boundary of the

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[4] Strictly speaking, θ is the geocentric angle between the satellite subpoint and the limit of the circle of coverage. However, it is convenient to talk about the arc on the surface of the earth which is subtended by this angle as the "radius" θ.

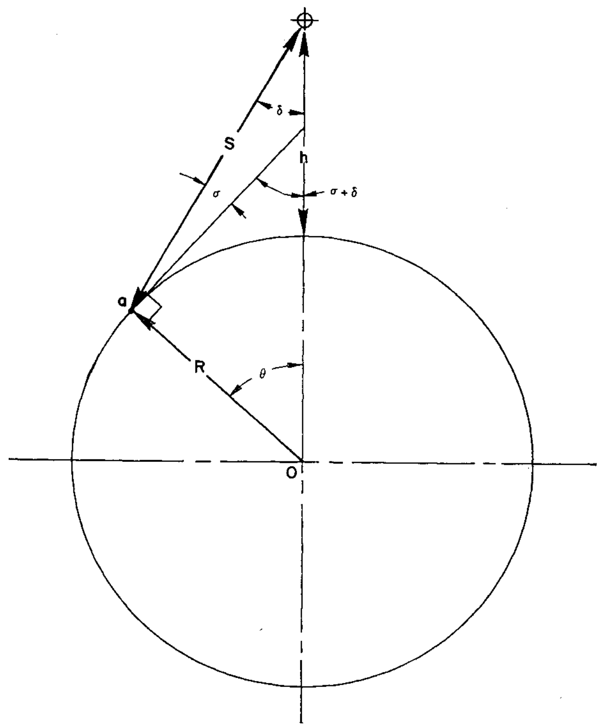


Fig. 1 Earth coverage of a single satellite

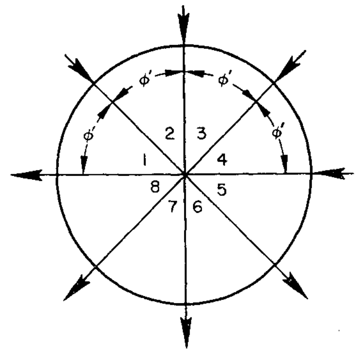


Fig. 2 Symmetrical polar network

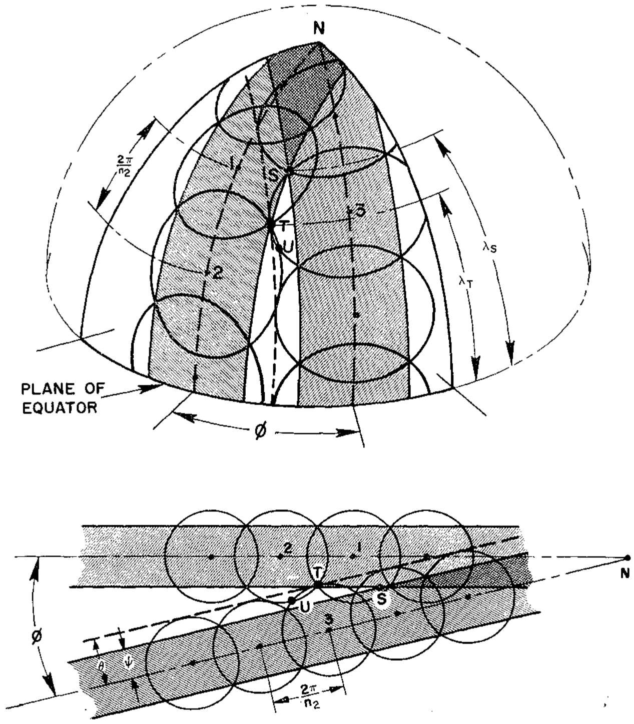


Fig. 3 Network coverage characteristics

detection areas define the band of continuous coverage having a width of 2ψ. Fig. 3 shows two such bands projected as solid lines on the surface of a quadrant of the earth for adjacent polar orbital planes. The intersection of these bands at point S defines the minimum latitude λ_S of continuous coverage for a symmetrical polar network. In such an arrangement of satellites there is no need to synchronize the satellite positions in one orbital plane with those in adjacent planes.

Optimum nonsymmetrical network

If the motion of the satellites in adjacent planes of the symmetrical satellite set shown in Fig. 2 is considered (arrows denoting direction of motion), note that in sections 2, 3, 4, 6, 7, and 8 the motion of adjacent satellites is co-rotational, whereas in sections 1 and 5 the motion is counter-rotational. Thus, if the satellites are synchronized in the sections where the motion is co-rotational, the latitude of minimum coverage for that section is defined by point T on Fig. 3.[5] Point T can be uniquely defined by the intersection of the boundary of continuous coverage of one orbital plane with the boundary of maximum coverage (dashed line) of the adjacent plane. However, in the sections where the motion is counter-rotational, it is impossible to synchronize the motion, and the latitude of minimum coverage is still defined by point S of Fig. 3. Figs. 4 and 5, which are plane sketches (lines of longitude drawn parallel rather than convergent) of the conditions at point T, are included only to illustrate why it is impossible to synchronize the motion of satellites traveling in opposite directions. It is seen that, as the satellites pass each other moving in opposite directions, small areas of width $\Delta\phi$ periodically open up in the band. Thus, the conditions in these two counter-rotational sections determine the design requirements of the symmetrical satellite network, whereas the remaining sections are, in effect, overdesigned. That is, if the satellites for the co-rotational sections were to be synchronized, the minimum latitude of continuous coverage obtained in these sections would actually be lower than that in the counter-rotational sections.

This leads to the concept of a nonsymmetrical arrangement of orbital planes as shown in Fig. 6. A symmetrical arrangement of satellites within each plane and synchronization between the satellites of adjacent planes is required. The angles ϵ and ϕ would be so chosen that the minimum latitude of continuous coverage would be the same for all sections. Such a network arrangement will permit lower orbital altitude than the symmetrical network for continuous coverage of a specific area for a fixed number of satellites and orbital planes.

The derivation of relations and the method of solution for the nonsymmetrical satellite system is included in the Appendix.

Comparison of Methods

Figs. 7 and 8 illustrate the effectiveness of the synchronized, nonsymmetric, orbital technique in reducing the satellite requirements from those of Ref. 1. The number of satellites required as a function of altitude and the corresponding slant range for the case $\sigma = 0°$ and $\lambda_T = 35°$ are considered. Significant improvements are evident in both respects. For the eight satellite networks, the orbital altitude may be decreased from 2528 naut miles (from the symmetrical orbit method of Ref. 1) to 2074 naut miles (using the proposed non-

[5] It may be argued that point U on Fig. 3 is, in fact, the point of lowest coverage. This is true for the positions shown; however, visualizing satellites 1, 2, and 3 an instant later than shown and moving in a southerly direction, it is apparent that a small opening will develop between the scan traces in the vicinity of point T. Therefore, point T is indeed the lowest point for continuous coverage.

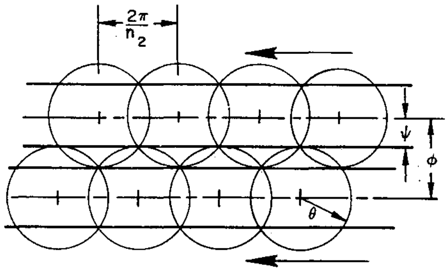


Fig. 4 Synchronized orbits

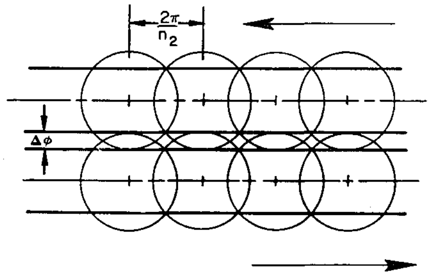
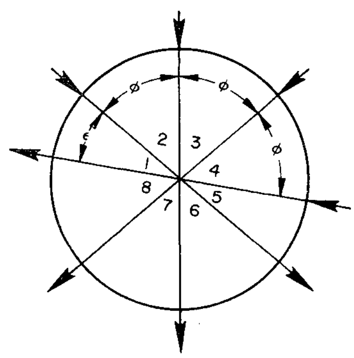


Fig. 5 Nonsynchronized orbits

Fig. 6 Nonsymmetrical polar network



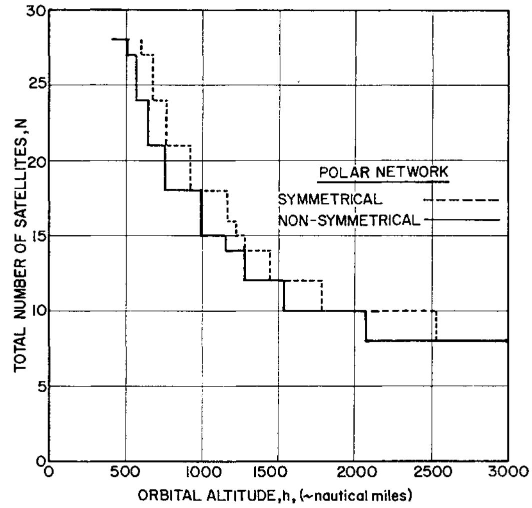


Fig. 7 Minimum number of satellites required for continuous zonal coverage ($\lambda_T = 35°$, $\sigma = 0$)

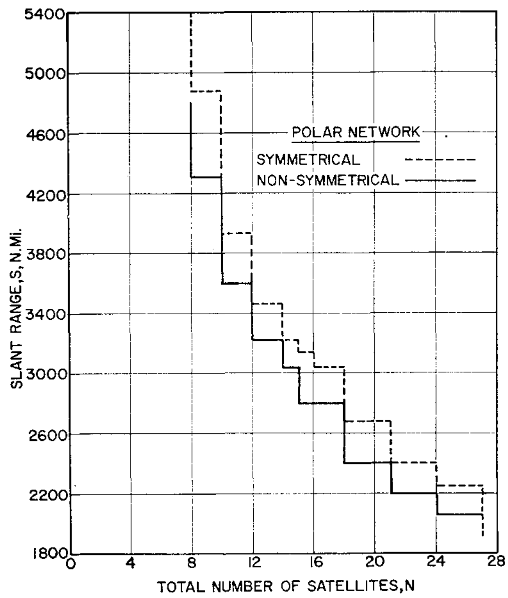


Fig. 8 Slant range for continuous zonal coverage ($\lambda_T = 35°$ $\sigma = 0$)

symmetrical technique). The corresponding decrease in slant range is from 4877 to 4309 naut miles.

Concluding Remarks

Significant reductions in orbital altitude and slant range can be obtained by synchronization of the satellites in adjacent polar orbits. The synchronized, nonsymmetric, orbital technique provides an optimum solution to polar satellite network design for continuous coverage.

The improvements obtained with a nonsymmetrical network for cases other than $\lambda_T = 35°$ are not shown in this paper. Although, in all probability, the level of improvement will vary somewhat with λ_T, the improvement shown for $\lambda_T = 35°$ is considered representative of that obtainable for all practical values of λ_T. In addition, the analysis has been limited to the special case of polar orbits. The nonsymmetrical concept discussed herein could be extended to inclined orbits. It is reasonable to assume that similar benefits could be obtained.

Appendix: Derivation of Relations and Method of Solution for Nonsymmetrical Satellite System

1 For the co-rotational segments, reference is made to Fig. 9. It is seen that the three satellites located at points A, B, and C have fields of view θ which intersect at point T, located at latitude λ_T. The latitudes of the three satellites are designated λ_1, λ_2, λ_3, respectively. Since it is desired to maximize the incremental longitude β between points T and C, the great circle arc TC must be perpendicular to the arc of longitude OC. The arc DT is a perpendicular to longitude OB and bisects arc AB. The derivation of the equations holds only in that region in which satellites 1, 2, and 3 are on the same "side" of the pole and thus is limited to $\theta \leqslant 90° - \lambda_T$. For fields of view θ greater than this, redundant coverage exists.

From spherical triangles DTB and DTO, it is seen that

$$\sin\lambda_T = \sin\left(\frac{\lambda_1 + \lambda_2}{2}\right)\left\{\frac{\cos\theta}{\cos[(\lambda_1 - \lambda_2)/2]}\right\} \qquad [A1]$$

but, since

$$\lambda_1 - \lambda_2 = 2\pi n_1/N \qquad [A1a]$$

$$\sin\lambda_T = \sin\left(\lambda_1 - \frac{\pi n_1}{N}\right)\frac{\cos\theta}{\cos(\pi n_1/N)} \qquad [A2]$$

$$\alpha = \sin^{-1}\frac{\sin(DT)}{\cos\lambda_T} = \sin^{-1}\left\{\frac{\{1-[\cos^2\theta/\cos^2(\pi n_1/N)]\}^{1/2}}{\cos\lambda_T}\right\} \qquad [A3]$$

From spherical triangle CTO

$$\sin\lambda_T = \sin\lambda_3\cos\theta \qquad [A4]$$

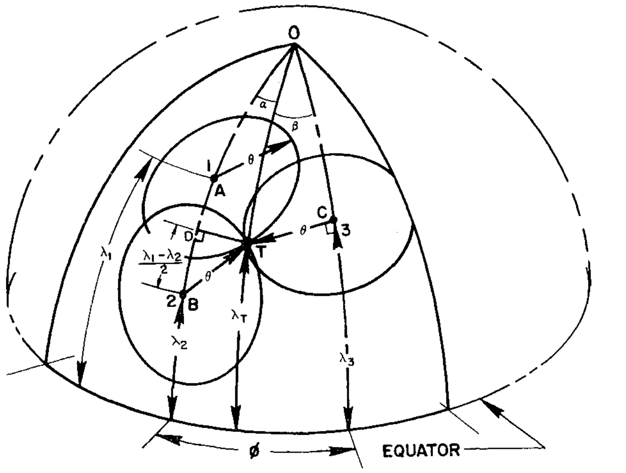


Fig. 9 Synchronization of satellites in adjacent orbital planes

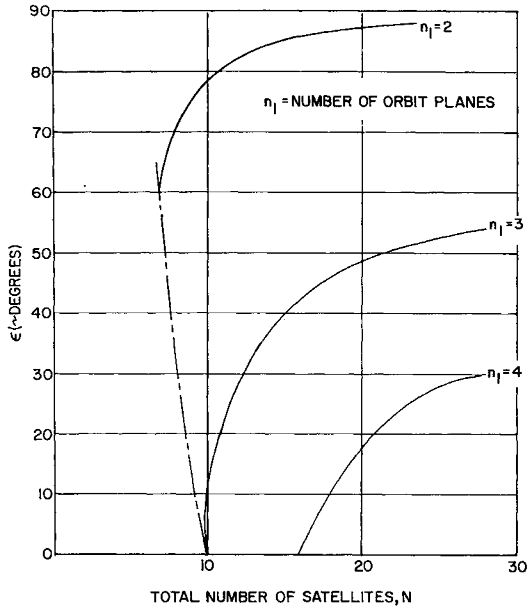


Fig. 10 Incremental longitude between counter-rotational satellite planes ($\lambda_T = 35°$)

$$\beta = \sin^{-1}(\sin\theta/\cos\lambda_T) \tag{A5}$$

Since $\phi = \alpha + \beta$,

$$\phi = \sin^{-1}\left\{\frac{\{1 - [\cos^2\theta/\cos^2(\pi n_1/N)]\}^{1/2}}{\cos\lambda_T}\right\} + \sin^{-1}\frac{\sin\theta}{\cos\lambda_T} \tag{A6}$$

2 For the counter-rotational segments, Ref. 1 shows that

$$\cos\theta = \cos(\pi n_1/N)\cos\{\sin^{-1}[\sin(\epsilon/2)\cos\lambda_T]\}$$

or

$$\epsilon = 2\sin^{-1}\left\{\frac{\{1 - [\cos^2\theta/\cos^2(\pi n_1/N)]\}^{1/2}}{\cos\lambda_T}\right\} \tag{A7}$$

3 Since any 180° segment of longitude must be made up of $(n_1 - 1)$ angles ϕ and one angle ϵ,

$$\pi = \epsilon + (n_1 - 1)\phi \tag{A8}$$

4 Eqs. [A1a–A8] are a set of eight simultaneous equations of eight unknowns (λ_1, λ_2, λ_3, ϕ, ϵ, α, β, θ). The equations cannot be solved in closed form for the angle θ. However, the solution may be accomplished by solving for θ as a function of n_1 and N, as outlined below.

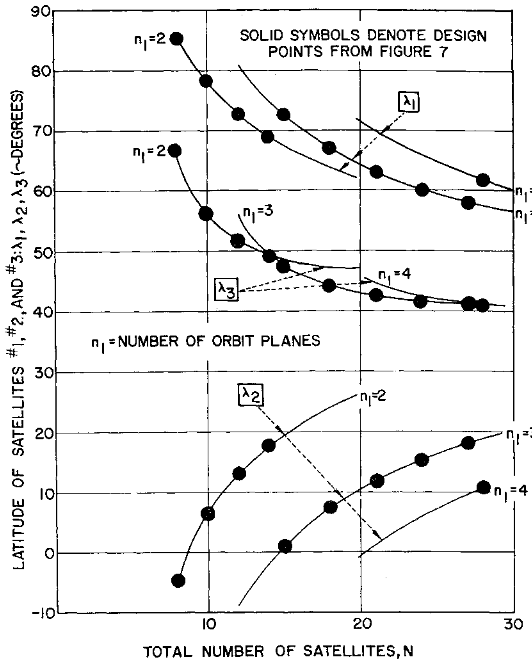


Fig. 11 Satellite latitude spacing requirements for synchronization ($\lambda_T = 35°$)

Combining Eqs. [A7] with Eqs. [A3, A5, and A8],

$$\sin\frac{\pi n_1}{N} = \left\{\frac{\sin^2\{[2\pi - \epsilon(n_1+1)]/2(n_1-1)\} - \sin^2(\epsilon/2)}{(1/\cos^2\lambda_T) - \sin^2(\epsilon/2)}\right\}^{1/2} \tag{A9}$$

Example of Solution, Special Case $\lambda_T = 35°$

A plot of ϵ as a function of N and n_1 from Eq. [A9] is shown in Fig. 10.

The solution of the equations has now been simplified to these steps:

1 For any n_1 and N, from Fig. 10 obtain ϵ.
2 Obtain ϕ from Eq. [A8].
3 Obtain θ from Eq. [A7].
4 Obtain λ_3, λ_1, and λ_2 from Eqs. [A4, A2, and A1a], respectively.
5 From Eq. [1] obtain altitude.

The plot of altitude h as a function of the number of satellites N for a specified number of orbital planes n_1 may then be made as suggested in Ref. 1, and the optimum arrangement of n_1 and n_2 may be determined from it. This procedure has been followed for the specific case of $\lambda_T = 35°$, $\sigma = 0°$, and the minimum required number of satellites as a function of orbital altitude is that previously presented on Fig. 7. The latitudes λ_1, λ_2, and λ_3, required at the instant the fields of view intersect, are shown on Fig. 11.

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